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A Novel Workflow to Model Permeability Impairment through Particle Movement and Deposition in Porous Media

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Abstract This article presents a practical transfer function type solution to a complex problem in which variations in a number of parameters can be taken into account. A new mathematical model, which is based on mass balance transfer function of particles movement/retention in porous media, has been derived. It is used to predict permeability reduction as a function of time. The linear forms as well as the radial forms of the model are described. Although the differential equations derived are similar to the general form of diffusion–convection equations, the marked difference is the suitability of the model, for being applied for variation of parameters, such as particle concentration in the fluid, injection rate, density of solid particles, against the depth and time of invasion. This transfer function has been solved, and the results of the simulation run agree reasonably well with the experimental damage data obtained in laboratory. Owing to its simplicity, this model is more practical to describe permeability reduction for the flow of suspended particles in porous media.

Keywords Permeability · Formation damage · Transfer function · Particle deposition · Porosity · Porous media · Mathematical model · Laplace transform

List of Symbols

A Area (m^2)
C Mass solid concentration (ppm)
C_d Constant

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K Permeability (m^2)
L Length (m)
M Model parameter (s^{-1})
m Mass of suspension particle (kg)
p Pressure (N/m^2)
q Volumetric flow rate (m^3/s)
t Time (s)
V Volume (m^3)
n Number of elements
ϕ Porosity
λ Trapping efficiency
λ_c Filtration coefficient
λ_0 Initial filtration coefficient
σ_v The volume of particles deposited per unit volume of the bed
ν Approach velocity
μ_1 Viscosity of the carrier liquid (kg/m s)
ρ Fluid density (kg/m^3)
τ Time constant

Subscripts

o Core inlet, stagnant or initial
b Bulk
i Initial or inlet
in Inlet
out Outlet
j Order of elements

1 Introduction

The phenomenon of solid particle invasion and occupation of the porous medium is a subject that is encountered in reservoir engineering applications. Numerous investigators study the process of solid particle invasion in a porous medium, related to deep bed filtration. The literature abounds with a number of theoretical studies and different approaches at mathematical modeling of permeability alteration due to solid particle invasion and capturing a porous medium most notably the application of effective medium theory and percolation theory in two- and three-dimensional capillary networks. A mathematical model is obviously necessary for theoretical explanation of the phenomenon of solid particle invasion into a porous medium. An attempt to describe the phenomenon of formation damage caused by solid particle invasion, and capturing a porous medium and the subsequent permeability alteration, is one of the reasons to develop a new mathematical model in this study.

Development of phenomenological model to describe the particle invasion in porous media was started at the beginning of past century. Iwasaki (1937) proposed three basic formulas governing the flow of particles in sand filters as follows:

$$\left(\frac{dC_v}{dx}\right) = -\lambda_c C_v \tag{1}$$

Table 1 Relationship between local particle retention and local permeability impairment

Investigators	Equations	Comments
Gruesbeck and Collins (1982)	$\frac{K_\mathrm{P}}{K_\mathrm{i}} = \exp\left(-a\left(\sigma_\mathrm{s}^\mathrm{p}\right)_\mathrm{p}^4\right)$	Empirical for pluggable paths, a is constant
	$\frac{K_{np}}{K_i} = \left[1 - b\left(\sigma_\mathrm{s}^\mathrm{p}\right)_{np}\right]^{-1}$	Empirical for non-pluggable paths, b is a constant
Soo and Radke Clayton (1986)	$\frac{K}{K_i} = 1 - \beta\frac{\delta}{\phi_0}$	β is the average flow restriction parameter
	$\frac{K}{K_i} = (1 - C_1 t)^2$	Gradual pore blocking $C_1 = \frac{\rho_\mathrm{p}^\mathrm{f}\alpha}{\rho_\mathrm{p}\mathrm{L}}$
Wojtanowicz et al. (1987, 1988)	$\frac{K}{K_i} = 1 - C_2 t$	Single pore blocking (screening) $C_2 = \frac{6Q\rho_\mathrm{p}^\mathrm{f}C_\mathrm{s}A^2}{\pi C_1 \mathrm{d}^3\rho_\mathrm{p}}$
	$\frac{K}{K_i} = \frac{1}{1+C_3 t}$	Cake forming (straining) C_3 is constant
Khilar and Fogler (1983, 1987)	$\frac{K}{K_i} = \left[1 - B\frac{\sigma_\mathrm{s}^\mathrm{p}}{\left(\sigma_\mathrm{s}^{*\mathrm{p}}\right)_i}\right]^2$	Theoretical model based on the Hagen–Poiseuille flow through pore throat, B is a parameter dependent on the characteristic
Civan et al. (1990)	$\frac{K}{K_i} = \left(\frac{\phi}{\phi_i}\right)^3$	Power law assumption
Rochon et al. (1996)	$\frac{K}{K_i} = \exp\left[\beta\left(\phi - \phi_i\right)\right]$	Linear relationship between the logarithm of permeability and porosity, β is relationship coefficient

$$\lambda_\mathrm{c} = \lambda_0 + C_\mathrm{d}\sigma_\mathrm{v} \tag{2}$$

$$\frac{\mathrm{d}\sigma_\mathrm{v}}{\mathrm{d}t} = \nu\frac{\mathrm{d}C_\mathrm{v}}{\mathrm{d}t} \tag{3}$$

The first equation gives the degree of particles deposition, proportional to their concentration at the sand filter inlet. The filtration efficiency is exhibited by the filtration coefficient, λ_c. The second equation relates the filtration coefficient to volume of the particles deposited per unit volume of the bed. The third equation represents the mass balance of particles in the system. The filtration coefficient, λ_c, plays a very important role and varies with the filter bed condition in the filtration process. The determination of this coefficient has been the subject of numerous investigations in deep bed filtration.

After Iwasaki, several researchers presented new models or modifications of previous models, to describe the phenomena of the deep bed filtration. A summary of the expressions for the coefficients is presented in Table 1.

Investigators were interested in deep bed filtration coefficient concerned with solid–liquid separation and particle deposition in porous media. However, there has not been much convincing evidence of their usage in formation damage, based on deep bed filtration.

There are six reported modeling studies concerning the in-situ generation and migration of fines in porous media (Gruesbeck and Collins 1982; Khilar and Fogler 1987; Soo and Radke Clayton 1986; Wojtanowicz et al. 1988; Civan et al. 1990; Rochon et al. 1996). In each of these studies simplified assumptions are made and the equations are solved to describe the reduction in permeability due to migration of the fine particles in porous media.

Gruesbeck and Collins (1982) addressed the problem of hydrodynamically induced fine migration. For colloidally induced fines migration, Khilar and Fogler (1983) used a mass balance approach, while Sharma and Yortsos (1987) have used a population balance and pore closure approach.

A quantitative model for the process of formation permeability alteration due to particle invasion and in-situ fine mobilization, using the principles of deep bed filtration and chemical reaction kinetics, has been proposed by Wojtanowicz et al. (1988). Three mechanisms of formation pore blocking have been recognized and analyzed in this model with regard to particles invasion, which are gradual pore blocking, single pore blocking (screening), and cake formation (straining).

Civan et al. (1990) presented a relationship between the initial permeability and the damaged permeability as a function of altered porosity (due to solid particles invasion) and initial porosity.

Song and Elimelech (1992, 1994) developed a theoretical model for the dynamics of colloid deposition in porous media by considering both macroscopic and microscopic theories of particle deposition. This model properly describes the dynamic behavior of particle deposition.

Rochon et al. (1996) presented a relationship between the initial permeability and the damaged permeability as a porosity exponential function.

2 Theoretical Basis and Model Assumptions

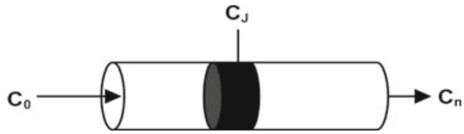
The mathematical expressions derived in this study have been obtained by conceptual representation of an actual porous medium, using a porous model based on the following fundamental and general assumptions:

1. Solid particles are uniformly suspended in an incompressible fluid.
2. The porous medium is homogeneous and isotropic.
3. The porous medium contains a large number of pore bodies, which are interconnected by pore throats which sizes are log-normally distributed.
4. The interaction forces between the glass beads and the suspension solids are not accounted for.
5. The fluid containing solid particles is injected into a porous medium at a constant flow rate.
6. The model is quite simplistic and does not consider microscopic aspects of particle deposition, hydrodynamic dispersion, and the detailed mechanisms of particle deposition and detachment.

3 Formulation of the Model

Consider an element of the porous medium of length Δx and cross-sectional area A with effective initial porosity, ϕ_i (Fig. 1). According to the fundamental law of mass conservation of solid particle, a solution with a concentration of C_0 that passes through a porous medium exits with the concentration of C_n.

Fig. 1 An element of the porous medium



The material balance relation for the particles in the fluid stream, flowing through an element of porous medium can be represented by the following expression:

Solids input rate − Solids output rate = Solids accumulation rate

The mass rate balance equation can be written as:

$$m_{\text{in}} - m_{\text{out}} = \frac{\mathrm{d}m}{\mathrm{d}t} \tag{4}$$

By expressing the equation mathematically, we can write:

$$Q\left(C_{j+1} - C_j\right) = V\frac{\mathrm{d}C_j}{\mathrm{d}t} \tag{5}$$

Based on the operating conditions observed in the experiments, the pore spaces of the porous medium are occupied by saturating fluid. Initially, the concentration of solid particles is set to zero.

Using Laplacian transform, we have:

$$\frac{C_{j+1}}{C_j} = 1 + \tau s \tag{6}$$

In which $\tau = \frac{V}{Q}$, and is called the time constant of the system. For the inlet element we have:

$$C_1 = C_0(1 + \tau s) \tag{7}$$

$$\frac{C_j}{C_o} = (1 + \tau s)^j \tag{8}$$

And evaluating C_n from Eq. 8 we will have:

$$\frac{C_n}{C_o} = (1 + \tau s)^n \tag{9}$$

Equation 9 is the Laplacian transform, which relates outlet concentration to the inlet concentration. Above-mentioned equations only account for concentration through packed bed. To evaluate porosity through packed bed, during flowing of unclean water through the pack, the following new model has been developed.

4 A New Model for Evaluation of Porosity

From definition of porosity, we can write:

$$\phi_j = \phi_i - \frac{\text{volume of particles}}{\text{Bulk volume of elements}} \tag{10}$$

$$\phi_j\left(t\right) = \phi_i - \frac{qtC_j}{60 \times 10^6 V_{bj}\rho_\lambda} \tag{11}$$

$$M = \frac{q}{60 \times 10^6 V_{bj}\rho_\lambda\phi_{\mathrm{i}}} \tag{12}$$

Arranging Eq. 11 and inserting M into it gives:

$$\frac{\phi_{j(t)}}{\phi_i} = \left(1 - MC_j t\right) \tag{13}$$

To calculate overall porosity, we use the definition of the porosity for total block:

$$\phi_{\text{total}} = \phi_{\text{e}} = \frac{\sum\left(\phi_{\text{j}} V_{bj}\right)}{\sum V_{bj}} \tag{14}$$

By knowing that $\sum \mathrm{V}_{bj} = n\mathrm{V}_{bj}$ and simplifying the Eq. 14 we have:

$$\frac{\phi_e}{\phi_i} = \left(1 - \frac{Mt \sum C_j}{n}\right) \tag{15}$$

Using permeability porosity correlation of Civan et al. (1990) equation, we can write:

$$\frac{K_e}{K_i} = \left(1 - \frac{Mt \sum C_j}{n}\right)^3 \tag{16}$$

And

$$\frac{K_j}{K_i} = \left((1 - MC_j t\right)^3 \tag{17}$$

4.1 Derivation of Transfer Function of the Porosity and Permeability

In this part, we have expanded the Laplacian transform of porosity, to derive transfer function of the porosity and permeability. Starting by Eq. 13:

$$\phi_j(t) = \phi_i(1 - MC_j t) \tag{13a}$$

Arranging Eqs. 4–13 for porosity:

$$\phi_j(t) - \phi_i = -\phi_i MC_j t \tag{18}$$

Taking Laplace transform gives:

$$\phi_j(s)/\phi_i = 1/s - M\ \ell(C_j t) \tag{19}$$

If $f(t)$ is continuous, its Laplace transform is $f(s)$:

$$\ell\left[f(t)\right] = f(s) \tag{20}$$

And knowing

$$\ell\left[tf(t)\right] = -\frac{\mathrm{d}f(s)}{\mathrm{d}s} \tag{21}$$

Using Eqs. 19 and 21:

$$\phi_j(s)/\phi_i = 1/s + M\frac{\mathrm{d}C_j(s)}{\mathrm{d}s} \tag{22}$$

By differentiating $C_j(s)$ and inserting it into Eq. 22 a transfer function of the porosity can be written as:

$$\phi_j(s) = \phi_i\left(1/s + M\frac{\tau_j}{(1+\tau s)^{j+1}}\right) \tag{23}$$

In the same manner, the permeability transfer function can be written as:

$$K_j(s) = K_i\left(1/s + \frac{(M\tau_j)}{(1+\tau s)^{j+1}}\right)^3 \tag{24}$$

For the last element, porosity transfer function can be written as:

$$\phi_n(s) = \phi_i\left(1/s + M\frac{\tau_n}{(1+\tau s)^{n+1}}\right) \tag{25}$$

In the same manner, the permeability transfer function for the last element can be written as:

$$K_n(s) = K_i\left(1/s + \frac{(M\tau_n)}{(1+\tau s)^{n+1}}\right)^3 \tag{26}$$

4.2 Evaluating Pressure in Each Node

Since the flow rate during injection is assumed to be constant, using Darcy equation we can write:

$$q_j = q_{j+1} \tag{27}$$

$$K_j\Delta P_j = K_{j+1}\Delta P_{j+1} \tag{28}$$

Equation 28 can be arranged to the following form: (for $j \neq 1$)

$$P_{j+1} = \frac{K_j}{K_{j+1}}\left(P_j - P_{j-1}\right) + P_j \tag{29}$$

Knowing

$$\frac{K_j}{K_{j+1}} = \left[\frac{\left(\frac{K_j}{K_i}\right)}{\left(\frac{K_{j+1}}{K_\text{i}}\right)}\right] = \left[\frac{\left(1-MC_jt\right)}{\left(1-MC_{j+1}t\right)}\right]^3 \tag{30}$$

Equation 29 can be written as follows:

$$P_{j+1} = \left[\frac{\left(1-MC_jt\right)}{\left(1-MC_{j+1}t\right)}\right]^3\left(P_j - P_{j-1}\right) + P_j \tag{31}$$

These equations hold for $j \neq 1$, for $j = 1$ we can write:

$$P_2 = \left(\frac{Q\mu\text{L}}{K1A}\right) + P_1 \tag{32}$$

where A is the area of the element and μ is the viscosity of the water.

4.3 Solution of the Model

The model solution is obtained by means of Simulink software and MATLAB program (Version 5.3.2). A diagram that was used to solve the model is shown in Fig. 2 that subsystem numbers 1, 2, and 3 are the same blocks as shown in Fig. 3.

The subsystem in homogenous porous media has the same characteristics which were used here for simplicity. But in general, for introducing the heterogeneity of the porous media, these subsystems can be used with different characteristics. Other models use an idea that permeability reduction is not the function of the flow pattern. By changing the block in the

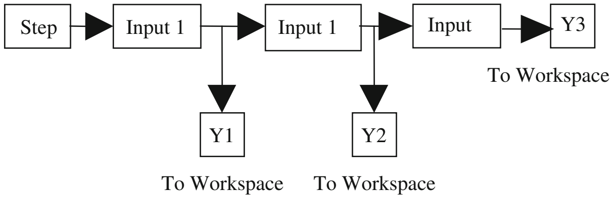


Fig. 2 Simulink diagram for solving the model

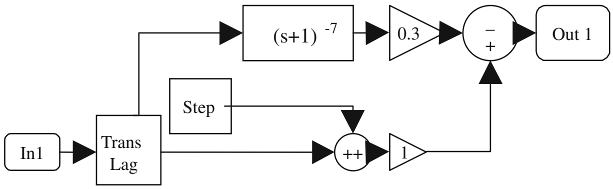


Fig. 3 Subsystem of input (1)

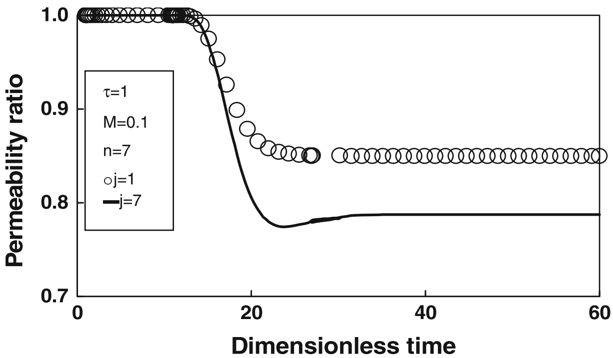


Fig. 4 Permeability ratio as a function of dimensionless time (Case 1)

subsystem number (2), this model can support all other types of flow patterns (in the laminar flow) as well as linear flow, for example radial flow.

This model can be used for any geometry because it does not depend on the shape of the porous media. Actual field with its actual boundary can be introduced to the Simulink software easily.

4.4 Validation of the Model

The model has been solved for two conditions ($\tau = 1$ and $M = 0.1$) in Fig. 4 and ($\tau = 1$ and $M = 0.2$) in Fig. 5, for the first element of the porous media ($j = 1$) and for the last one ($j = n = 7$). These figures illustrate a predicted permeability ratio versus dimensionless time t/τ. In order to compare the experimental data with data predicted by the model, Figs. 6 and 7 have been presented. A glance view gives this idea that in the front elements, prediction ability of the model is better than in the final ones. This is not the inability of the model, but is the online data substation of Simulink software. To avoid this effect, a transfer lag function has been inserted into the block diagram of the solution, as shown in Fig. 3.

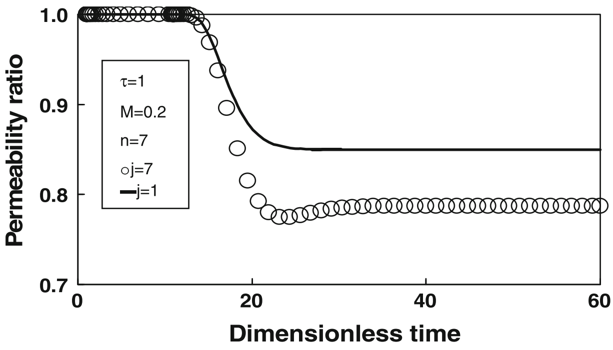


Fig. 5 Permeability ratio as a function of dimensionless time (Case 2)

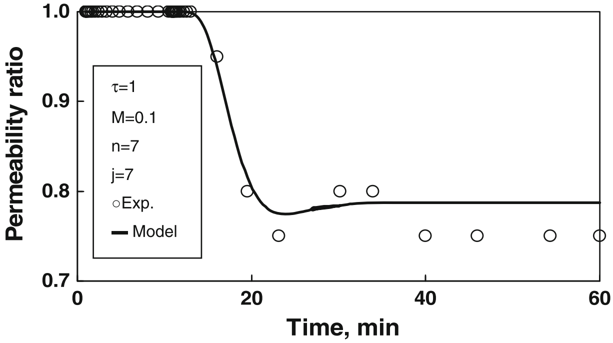


Fig. 6 Comparison of model prediction and experimental data for Case 1

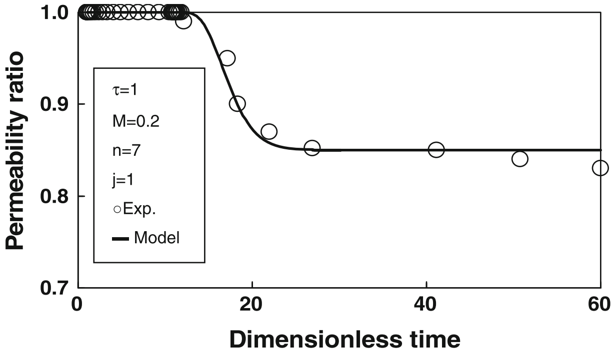


Fig. 7 Comparison of model prediction and experimental data for Case 2

This transfer function has an important role in the optimization of the quality of the model solution. Since the response of water injection into the porous media has been observed by the time lag, which equals to the time required to transfer from the frontal element to the last element or back front, this is necessary for the model to introduce the transfer lag function for each element. In this special solution, an exponential transfer lag function has been used. We find out that, this selection has enough reasons, in comparison with the other transfer lag functions.

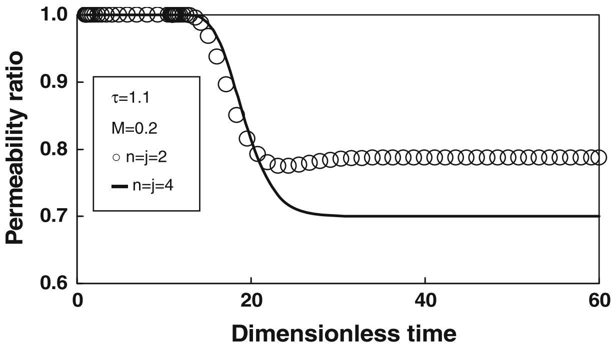


Fig. 8 Permeability ratio as a function of dimensionless time with considering time lag and the number of elements

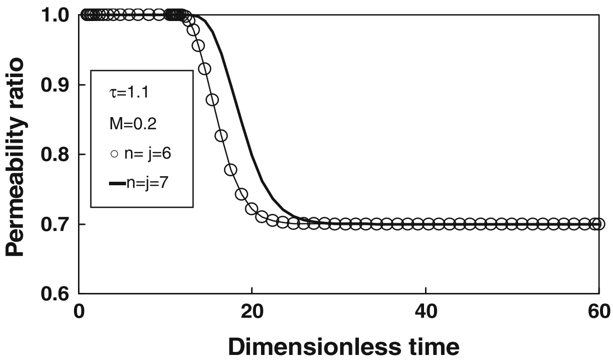


Fig. 9 Permeability ratio as a function of dimensionless time with considering time lag and the number of elements

Figure 8 is a result of a run with condition ($\tau = 1.1$ and $M = 0.2$), which compares two division of porous media ($n = j = 2$ and $n = j = 4$), which shows permeability ratio for the final elements. Owing to the large volume of the elements, the results are very apart from each other. But by increasing the number of the elements, we get a better solution for permeability.

Figure 9 shows a condition with higher value of n, the number of elements ($n = 6$ and $n = 7$). This is obvious that their results are similar but yet they do not fit completely. For doing this, we run the model for higher values again, as shown in Figs. 10 and 11. In Fig. 11 a complete fitting has been achieved. Solving Eqs. 31 and 32, and solution of permeability reduction can represent pressure and pressure drop profile. To evaluate these profiles MATLAB software has been used.

These profiles are presented here to show that, by using the proper transfer lag, critical concentration could be introduced to the model. Critical concentration, that was reported by Gruesbeck and Collins experimentally for the first time now, is supported by this new model. This critical concentration can be detected in Figs. 12 and 13.

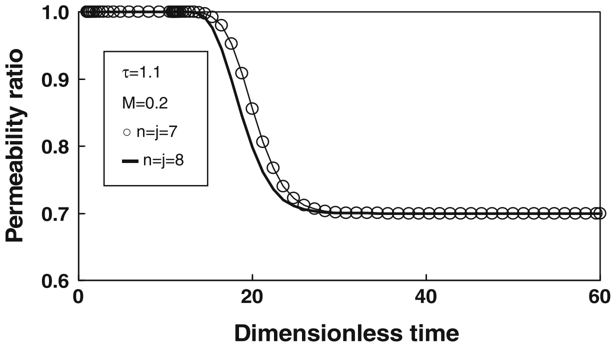


Fig. 10 Permeability ratio as a function of dimensionless time with considering time lag and the number of elements

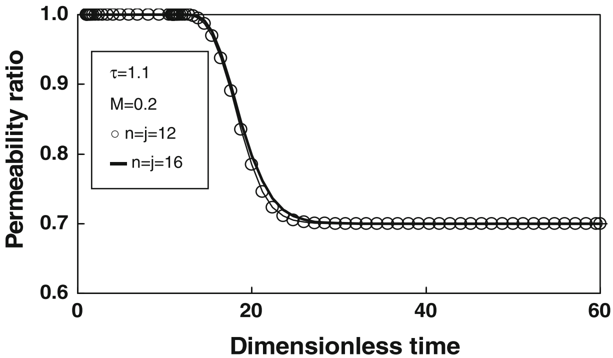


Fig. 11 Permeability ratio as a function of dimensionless time with considering time lag and the number of elements

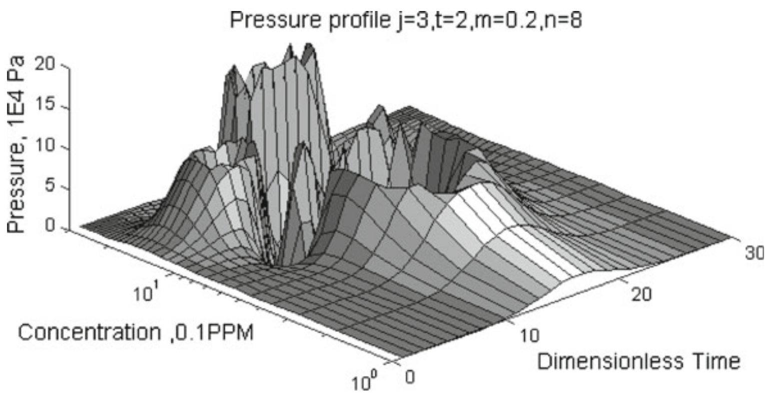


Fig. 12 Pressure Profile as a function of dimensionless time and concentration

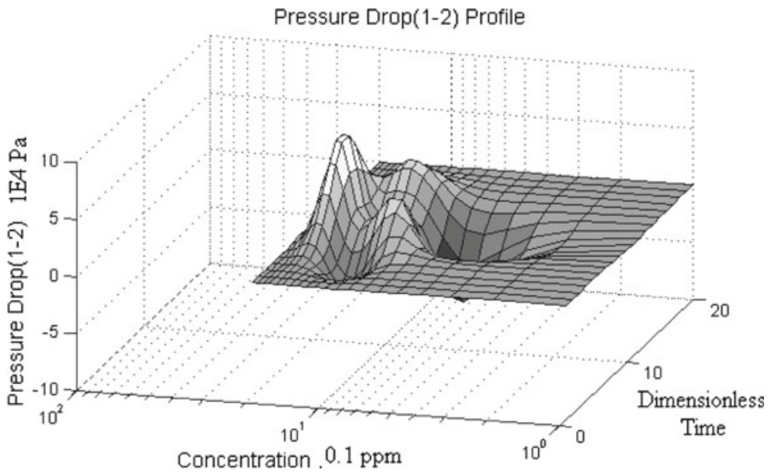


Fig. 13 Pressure Drop as a function of dimensionless time and concentration

5 Conclusions

In this article:

An analytical model for describing particle movement and deposition in porous media has been derived. Solution to the model has been provided by Simulink software.

The major characteristics of this model are as follows:

a. This model does not depend on the flow pattern and can be used for any type of flow including linear and radial.
b. Since the model does not depend on the shape of porous media, it can easily be used for any geometry.
c. One of the points that makes this model noticeably different from the previous ones presented in the literature is its ability to take into account several parameters such as particle concentration in the fluid and injection rate, against the depth and time of invasion.

The critical concentration concept that had previously been presented theoretically and experimentally now is supported by this new model and can be detected from pressure and pressure drop profiles.

The presented model is capable of simulating solid particle movement and deposition in the porous media. The modeling results obtained by running the phenomenological model demonstrated that this model could predict the trend of permeability damage due to the particle movement and deposition during injection.

Appendix A: Choosing the Transfer Lag Function

As mentioned, by considering a transfer lag in the model solution, we observe a better simulation. But the type of transfer function is highly affected by the geometry of the flowing fluid and flow regime in the porous media. This is the reason that we can say this model can bring the flow regime into account.

The importance of the transfer function was mentioned, but how we can choose a correct transfer lag function for the certain regime is considered here. We know that transfer lag has finite value at infinite time; thus, a Continuous function that has these characteristics can be tested for the model, such as:

$$f_l(t) = \frac{(1 - \tau t/2)}{(1 + \tau t/2)} \tag{A-1}$$

Or this function:

$$f_l(t) = \frac{\left(1 - \tau t/2 - \tau t^2/6\right)}{\left(1 + \tau t/2 + \tau t^2/6\right)} \tag{A-2}$$

But these functions have time restriction to use. This function is negative in this range:

$$t < 2/\tau \tag{A-3}$$

Consider the following transfer functions:

$$f_l(t) = e^{\frac{(1-\tau t/2)}{(1+\tau t/2)}} \tag{A-4}$$

$$f_l(t) = e^{\frac{\left(1-\tau t/2-\tau t^2/6\right)}{\left(1+\tau t/2+\tau t^2/6\right)}} \tag{A-5}$$

Positive transfer lag is achieved by using them. But which of them can be used is more complicated and practical to discuss about here, so a trial and error for checking the system for primary modeling is proposed.

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